Project 9
What are weak measurements?

Overview

This project explores a remarkable quantum effect: how a spin-$\frac{1}{2}$ particle, when weakly measured, can yield a result like 100. This paradox was introduced in a 1988 paper by Aharonov, Albert, and Vaidman. In a weak measurement, a quantum system is probed so gently that its state is hardly disturbed, and yet meaningful information can still be extracted — especially when both initial (pre-selected) and final (post-selected) states are specified.

Guiding Questions

- What does it mean to pre-select and post-select a quantum state?
- How do forward- and backward-evolving wavefunctions coexist in time?
- Can a quantum system be influenced by both its past and its future?
- How can a measurement result lie outside the eigenvalue spectrum?
- Why are weak measurements useful? What can they reveal that strong ones cannot?

The Core Phenomenon

Let a system be pre-selected in state $|\psi_i\rangle$ and post-selected in state $|\psi_f\rangle$. A weak measurement of observable A yields the **weak value**:

$$A_w = \frac{\langle\psi_f|A|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}.$$

This value can be much larger than any eigenvalue of A, or even negative or complex. Aharonov et al. showed that the weak value of a spin component can be 100 — not because the spin is actually 100, but because the pointer shift (after many weak measurements) reflects this amplified value.

Tasks

1. Conceptual Summary

Read Chapters 16 and 17 of *Quantum Paradoxes* by Aharonov and Rohrlich. Then, in your own words:

- Explain how weak measurements work in the two-state formalism.
- Summarize the “spin = 100” thought experiment.

2. Core Calculation

Let the system be:

- Pre-selected in: $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_z^A|+\rangle_z^B + |-\rangle_z^A|-\rangle_z^B\right)$
- Post-selected in: $|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_x^A|-\rangle_x^B + |-\rangle_x^A|+\rangle_x^B\right)$
- Operator: $P_{++}^{xx} = |+\rangle_x^A|+\rangle_x^B\langle+|_x^A\langle+|_x^B$

Compute the weak value:

$$(P_{++}^{xx})_w = \frac{\langle\Phi|P_{++}^{xx}|\Psi\rangle}{\langle\Phi|\Psi\rangle}.$$

Is the result negative? What does that mean for a pointer that couples to this operator?

3. Interpretation and Use

Write a short explanation or presentation addressing:

- How weak and strong measurements differ.
- What anomalous weak values suggest about intermediate quantum states.
- One modern application (e.g., signal amplification in quantum optics).

Outcome

You should come away understanding:

- The formalism and motivation of weak measurements.
- How paradoxical results (like spin = 100) arise.
- Why weak measurements provide unique access to the system's behavior between measurements.