Interpretations of Quantum Mechanics

Overview

1. Introduction
2. The Copenhagen Interpretation
3. The Pilot-wave theory
4. The many-worlds Interpretation
5. Spontaneous Collapse theory
6. Bell's Theorem

literature :

* Travis Norsen, Foundations of Quantum Mechanics

(an exploration of the physical meaning of QM)

other books :

* John Bell, The speakable and unspeakable in QM

* Jean Bricmont, Making sense of QM

* Heisenberg, Philosophy of physics
  Der Teil und das Ganze

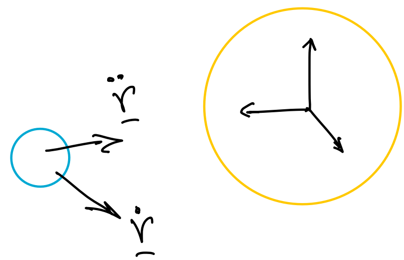
* Zeilinger, The message of the quantum

1. Introduction

1.1. Historical remarks

- Earth not a special place!

  → Galilean relativity



- $\underline{F}(\underline{r}) = m\ddot{\underline{r}}$ Newton

  Force

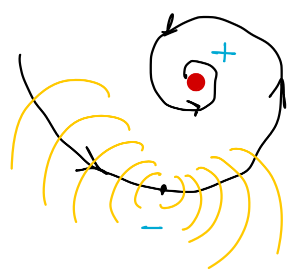
having $\left\{ \begin{array}{l} \ddot{\underline{r}} \\ \underline{r}(0) \\ \dot{\underline{r}}(0) \end{array} \right.$ we can calculate $\left\{ \begin{array}{l} \text{position } \underline{r}(t) \\ \text{momentum } \underline{p} = m\dot{\underline{r}} \end{array} \right.$

- Discovery of atoms

prediction of classical physics (mechanics & ED):

electron in atom emits light

while accelerating in nucleus's field.

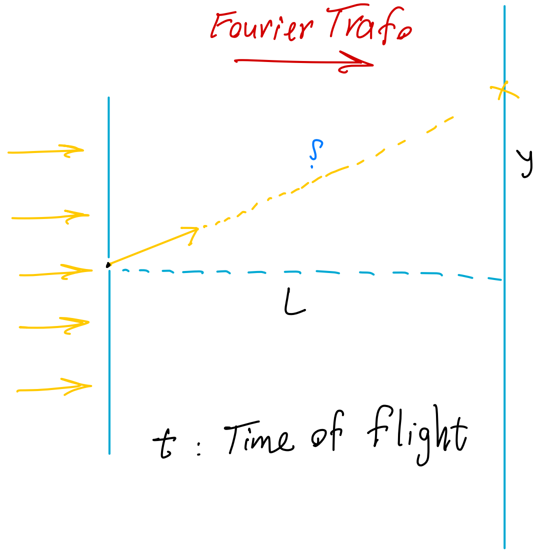


→ Atoms are not stable!

* Heisenberg's uncertainty principle

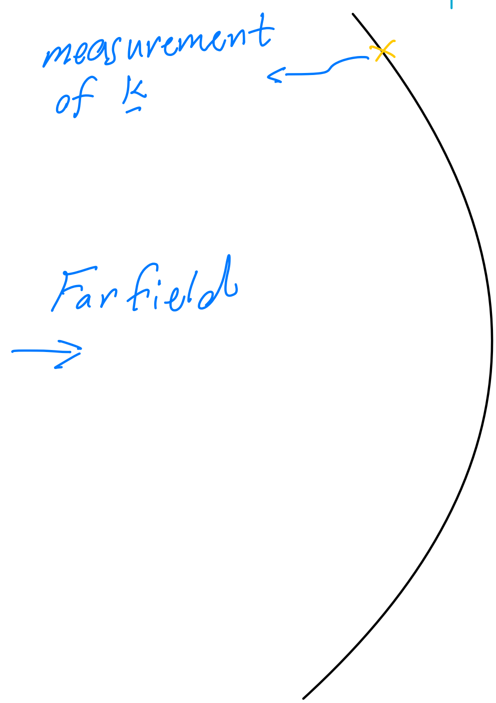
Simultaneous position and momentum of a particle cannot be adequately measured. How?

$$\Delta x \, \Delta p \geq \frac{h}{2\pi}$$



- how one measures momentum?

- principally momentum is the Fourier-transform of position.



Schrödinger's wave mechanics

$$i\hbar\,\partial_t \Psi(\underline{r},t) = \left(-\frac{\hbar^2}{2m}\Delta + V(x)\right)\Psi(\underline{r},t)$$

electron as a standing wave in the nuclei potential:

$$V(x) = k\frac{q}{r}$$

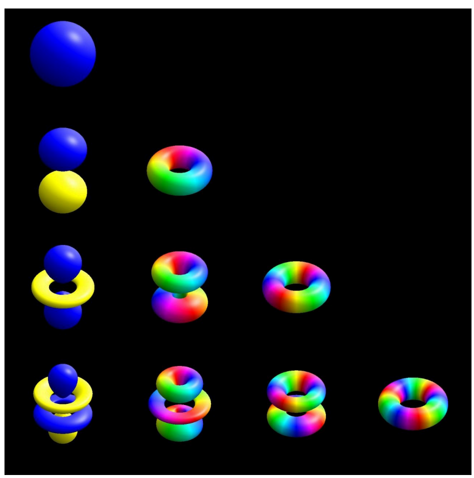
motion without trajectory (path)!

probability of finding the electron at $\underline{r}, t$

$$P(\underline{r},t) = |\Psi(\underline{r},t)|^2 = \Psi(\underline{r},t)\,\Psi^*(\underline{r},t)$$

* quantised Angular Momentum. $\hat{L} = \hat{r}\times\hat{p}$

– electronic orbitals instead of orbits.



Einstein-Heisenberg debate over the path of the particle.

* Der Teil und Das Ganze
W. H

if we can see the path of the electrons in cloudchamber, why can't they have paths inside atoms as well?

Einstein to Heisenberg
Berlin, 1926

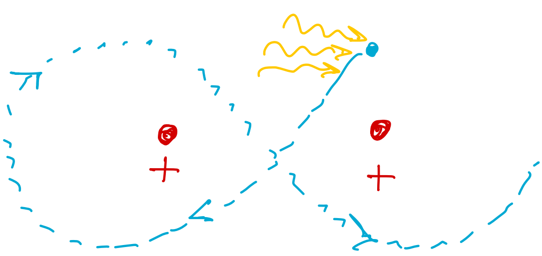
Exercise: think about a possible answer!
more to this in Chapter 2

debatable:

* Can one force the electron in e.g. He^+ to move on a quasi-classical path?

Yes! using ultra-short pulsed laser.

Thomson



Schrödinger - Bohr debate over Quantum Jumps

* Der Teil und Das Ganze W. H

S. How is the electron's behaviour during a transition between one quantised state to another in atom?

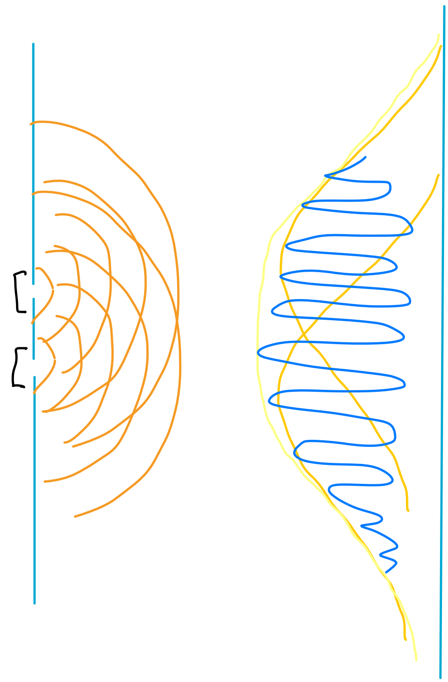
"If one has to stick to this damned quantum jumping, then I regret having ever been involved in QM!" Schrödinger to Bohr München Seminar 1926

Exercise: think about a possible answer!

Einstein - Bohr debate in chapter 2

Copenhagen Interpretation

Double Slit experiment



- wave fronts exert the slits in phase.

→ Interference happens

- Wave-particle duality:

There is a particle associated with the wave.

- particles go throw both slits.

- maybe particles scatter or interact ?

- send particles one by one : Interference keeps happening

→ Interference isn't a many particle effect.

→ Each particle interferes with itself !

Possible interpretation:

Particle seems to go through both Slits at once!

Introducing a new concept:

$\boxed{\text{Particle turns into a superposition of going through both slits.}}$

* Superposition: An entirely new kind of existence? Nobody can observe a particle exiting both slits simultaneously.

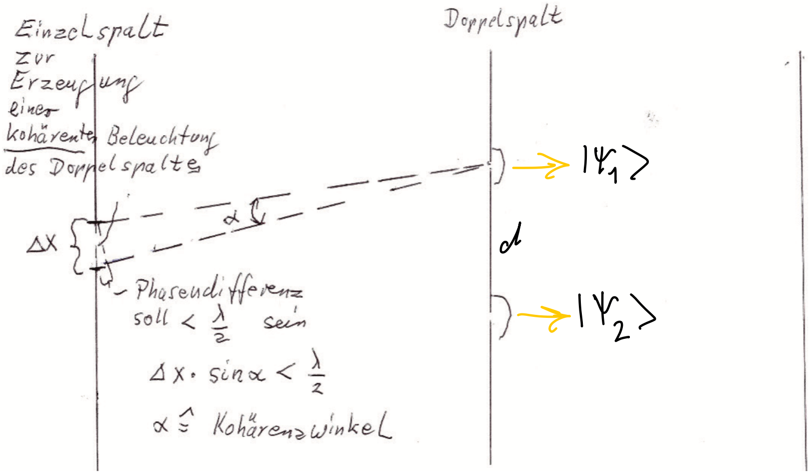
- but mathematically it's not a weird concept:

$\boxed{\text{If } \psi_i \text{ is a possible state of a system, e.g. a particle then any superposition } \psi = \sum c_i \psi_i \text{ also possible.}}$

- when the which-way information not available in the double-Slit setup:

→ Superposition of both ways → Interference

- Superposition in Double-Slit experiment:



Skript Atomphysik 1 Dörner 2021

$$|\Psi\rangle = |\Psi_1\rangle + |\Psi_2\rangle \quad \text{in Hilbert space}$$

$$\Psi_{\underline{k}}(\underline{r},t) = \left(A_1(\underline{r})\, e^{i\underline{k}\cdot(\underline{r} + \frac{d}{2}\underline{e}_y)} + A_2(\underline{r})\, e^{i\underline{k}\cdot(\underline{r} - \frac{d}{2}\underline{e}_y)} \right) \cdot e^{-i\omega t}$$

- Calculate $|\Psi(\underline{r},t)|^2 = \Psi(\underline{r})\,\Psi^*(\underline{r})$

Intensity of particles on the screen.

on the detection screen:

$$= \left(A_1(\underline{r})\, e^{i\underline{k}\cdot(\underline{r} + \frac{d}{2})} + A_2(\underline{r})\, e^{i\underline{k}\cdot(\underline{r} - \frac{d}{2})}\right)\left(A_1^*(\underline{r})\, e^{-i\underline{k}\cdot(\underline{r} + \frac{d}{2})} + A_2^*(\underline{r})\, e^{-i\underline{k}\cdot(\underline{r} - \frac{d}{2})}\right)$$

$$= |A_1(\underline{r})|^2 + |A_2(\underline{r})|^2 + \quad ,\ ,\ ,\ \text{Exercise}\ ,\ ,\ ,$$

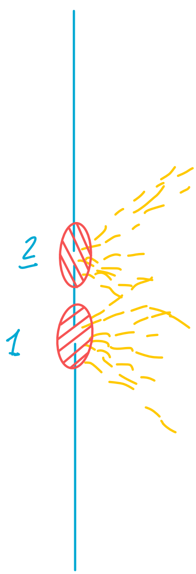
* How to know through which slit (which way?) did the particles actually go?

- Make the which-way information principally available (leakage of information) by relating the slits differently to an intrinsic property of the particle.

example: Putting perpendicular polarisers on slits.

:= putting which-way mark on particle.

result: No Interference!

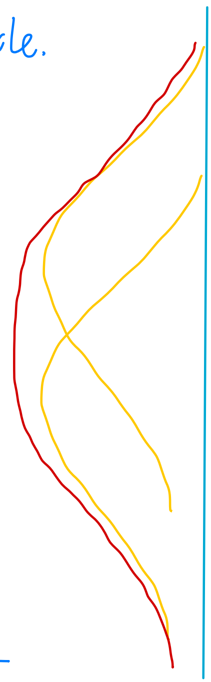


passed particle

Polarisation determined

if 45° : went through (1)

if 135° : went through (2)



We don't necessarily need to know the polarisation, it just needs to be principally possible to know in the new setup

* Did we just measure the which-way ?

- Yes, $|\psi\rangle = |\psi_1\rangle + |\psi_2\rangle$ reduced to $|\psi_1\rangle$ or $|\psi_2\rangle$

Particles go through one of the slits each time: no superposition ?

- Measurement in QM :

$$|\psi\rangle = \sum c_i \psi_i$$

Collapse or reduction of the State (: wave-function)

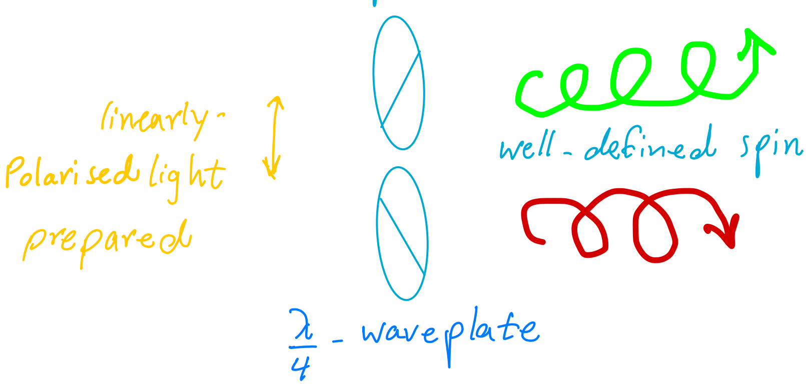
to one of the eigenstates. $|\psi\rangle \rightarrow |\psi_i\rangle$

with probability : $P_i = |c_i|^2$

- not necessarily : Explaining Measurement in terms of Entangled Superposition of the Particle-Apparatus

- more about Entanglement next page.

* Another example of which-way marker :



which-way mark can be put with a coherent change in the state.

* quantum entanglement (crash cource)

Consider the simplest quantum system:

spin $\frac{1}{2}$ particle: two possible states $\uparrow$ $\downarrow$

naming them $|0\rangle$ and $|1\rangle$ in 2d Hilbert space.

A system of two particles: $|0\rangle$ $\uparrow$ $|1\rangle$ $\downarrow$

not. Entangled

A B

each can be in superposition of $|0\rangle$ and $|1\rangle$

$$|A\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A + |1\rangle_A) \qquad |B\rangle = \frac{1}{\sqrt{2}}(|0\rangle_B + |1\rangle_B)$$

The wave function of both particles is:

$$|AB\rangle = \frac{1}{2}(|0\rangle_A + |1\rangle_A) \otimes (|0\rangle_B + |1\rangle_B)$$

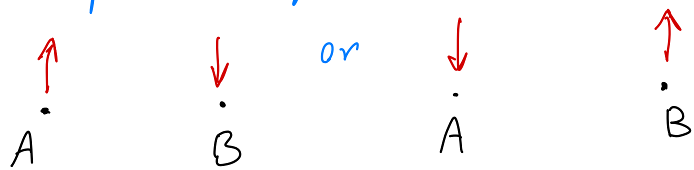
$$= \frac{1}{2}\left(|0\rangle_A |0\rangle_B + |0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B\right)$$

or $:= \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

- Entangled superposition:

Using the Conservation of Spin

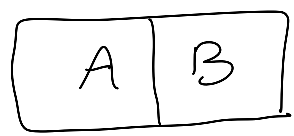
if the total spin of particles A and B is zero :



→ Entangled Stat : $|AB\rangle = \frac{1}{2}(|01\rangle + |10\rangle)$

Cannot be written as product-State:

$|AB\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$ Entangled state



compare with:

$$|AB\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |00\rangle)$$
$$= \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes (|1\rangle_B + |0\rangle_B)\right)$$
$$= |0\rangle_A \otimes \frac{1}{\sqrt{2}}(|1\rangle_B + |0\rangle_B)$$

product state

→ not entangled

if A and B entangled, we can't write a state for each subsystem alone.

Entangled superpositions don't exhibit Interference!

Example: Particle's wave function $|\psi\rangle$ going through the double-Slit DS, which-way information available

$$|DS, \psi\rangle = \underbrace{N}_{\text{normalising factor}} \left(|L\rangle|x\rangle + |R\rangle|y\rangle \right)$$

L, R Left - Right Slit

x, y horizontal, Perpendicular Polarisation

$$\langle DS, \psi | DS, \psi\rangle = |N|^2 \left(\langle x|\langle L| + \langle y|\langle R|\right)\left(|L\rangle|x\rangle + |R\rangle|y\rangle\right)$$

$$= |N|^2 \Big(\overbrace{\langle x|x\rangle}^{|\psi_1(\underline{r})|^2} \underbrace{\langle L|L\rangle}_{=1} + \overbrace{\langle y|y\rangle}^{|\psi_2(\underline{r})|^2} \underbrace{\langle R|R\rangle}_{=1} + \langle x|y\rangle \underbrace{\langle L|R\rangle}_{=0} + \langle y|x\rangle \underbrace{\langle R|L\rangle}_{=0} \Big)$$

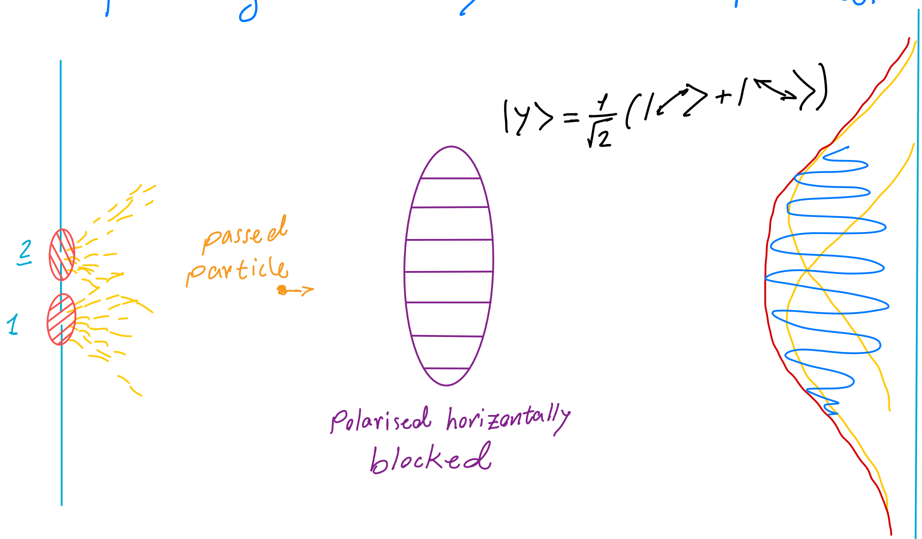
interference terms vanish

$$= |N|^2 \Big(\underbrace{|\psi_1(\underline{r})|^2 + |\psi_2(\underline{r})|^2}_{\text{Sums of the individual intensities}} \Big)$$

* Can we bring back the lost interference?

Yes! → Quantum Eraser

:= erasing which-way information after putting which-way mark on particle.



- Can we bring back the which-way info?

No! Erasure is irreversible.

- Can we erase the which-way info after particles hitting the screen?! Yes!

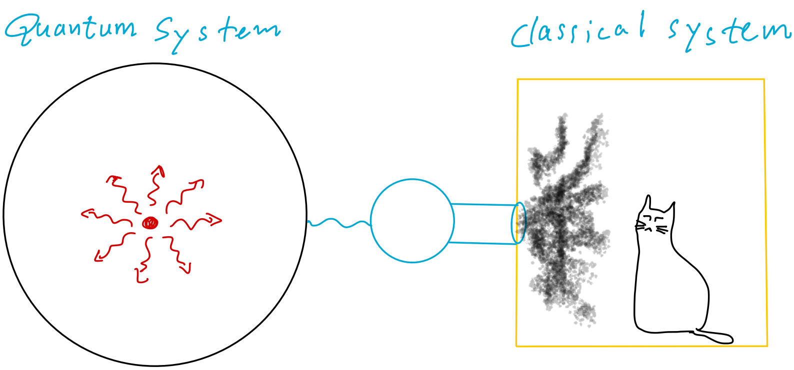
Delayed choice: using Entanglement (can be discussed)
Q-Eraser

- How would you interpret this?

1.2. Measurement Problem

- The entangled superposition of the quantum object and the measuring apparatus, turns the apparatus into a superposition of all possible outcomes. i.e. a superposition of macroscopic objects. This is ABSURD according to Schrödinger/Einstein.

- Schrödinger's Cat Gedankenexperiment:



- nobody has seen a blurry dead & alive mixture of cats. QM applied to macroscopic world
→ reductio ad absurdum

wrong:

$$|\text{Not Decayed}\rangle + |\text{Decayed}\rangle \qquad |\text{Alive}\rangle + |\text{Dead}\rangle$$

Correct state: Entangled state

$$|\text{Not Decayed}\rangle|\text{Alive}\rangle + |\text{Decayed}\rangle|\text{Dead}\rangle$$

1.3. Interpretations of QM are different – approaches to measurement problem

1.3.1. Copenhagen Interpretation (orthodox QM)

- keeps the measurement problem unsolved.

  measurement Pustulate in QM, Born's rule

- **The wave function gives all the Information** about the q-system!

  $|\psi(t)\rangle$

- The quantum-System described with QM while the measuring apparatus described classically.

- No Theory for measurement process itself.

- No clear, consistent, unified, literal way to describe quantum phenomena is possible!

- Moral: Don't try to find better concepts than the already existing classical concepts.

- Complementarity, Uncertainty-relations

1.3.2. de Broglie-Bohm pilot-wave theory

- particle or wave $\longrightarrow$ particle and wave
- particles move on trajectories governed by the wavefunction.
- Introduces a so called Hidden Variable and completes the usual QM to a classical deterministic theory. (Particles have well-defined paths)

Hidden variable: $X(t)$ actual position of particles

$$\psi(\underline{r},t) = R(\underline{r},t)e^{iS(\underline{r},t)}$$

$S(\underline{r},t)$ Phase of ψ

$R(\underline{r},t)$ modulus of ψ

1. $i\hbar\,\partial_t \psi = \hat{H}\psi$
2. $\partial_t X(t) = \frac{\hbar}{m}\underline{\nabla} S(\underline{r},t)$

- No division of the world into classical/Quantum.

- Solves the measurement problem!

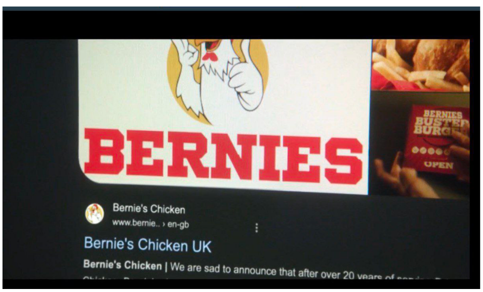
* Discussion of Contextuality Chapter $\underline{3}$

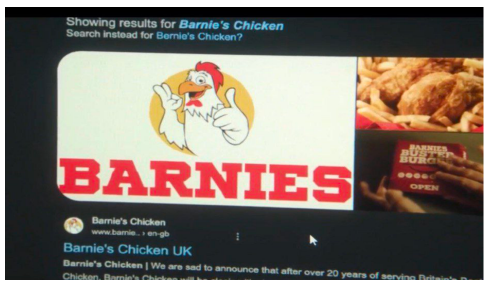
1.3.3. Interpretations that get rid of the measurement problem:

*** Many-worlds or Multiverse Theory:** Chapter 4

- wave function gives complete information.
- No division of the world into classical/Quantum.
- No external observation.
- Entanglement, Decoherence → Creation of parallel realities.

Everett





* Popular in Science-fiction, Black mirror 2025

*** objective collapse of the $\psi(r,t)$ Theories:**

- GRW (Spontaneous Collapse) Chapter 5
- Gravitation's role in the Collapse of the ψ
  Penrose.

...

1.4. Ontology

Definition: What exists? What is real?

Question: Are Superpositions real?

They exist, but we can't observe them!

Question: Is the wave function physically real?

Question: Are physical quantities e.g. Spin real independent of our observation?

Question: Can we define physical reality?

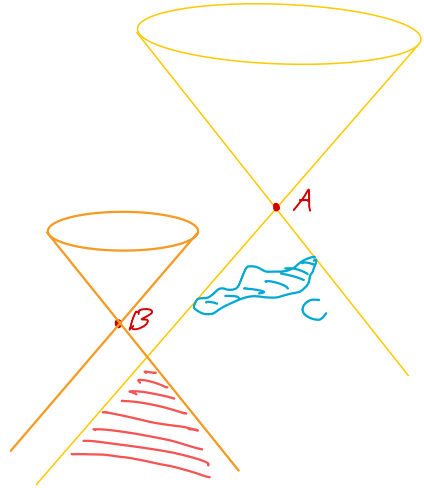
ERR: If the theory predicts it with 100% Probability without in any way disturb/changing the system.

Three positions: Griffith

- realist
- Orthodox Quantum theory (Copenhagen)

Agnostic

1.4. Locality chapter 6



Special relativity

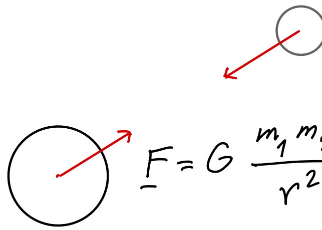
Bell's definition of locality:

$$P(A|C,B) = P(A|C)$$

Maxwellian Electrodynamics: $\Box \underline{A}(\underline{r},t) = 0$

per construct local!

Newtonian Gravity:

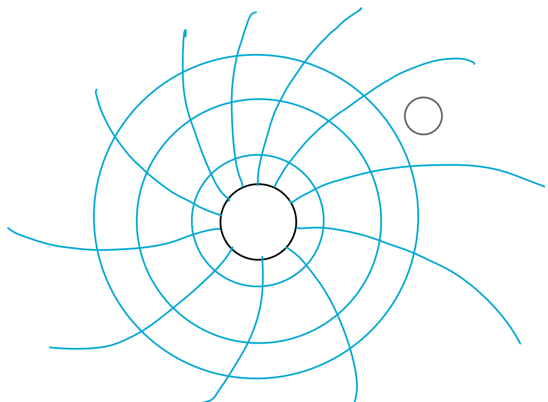


non-local

$$\underline{F} = G\frac{m_1 m_2}{r^2}\hat{\underline{r}}$$

Instantaneous

General relativity

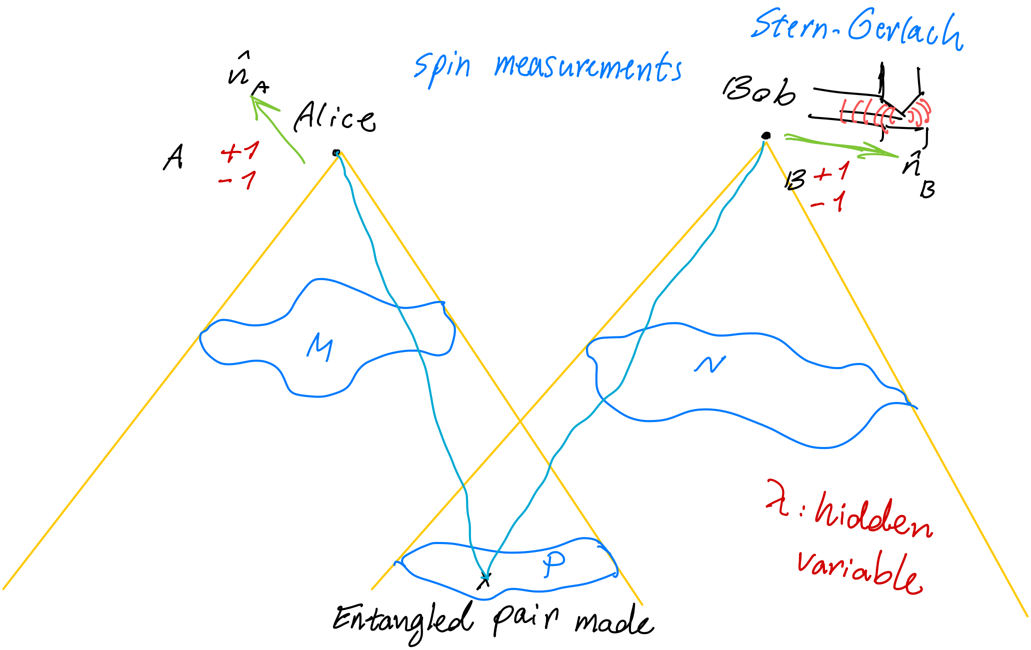


local

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

Einstein-Podolsky-Rosen Paradox (Bohm's version)

* had measurement outcomes existed prior to measurement?



$$\psi_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B\right)$$

Subsystems A and B don't possess a definite state.

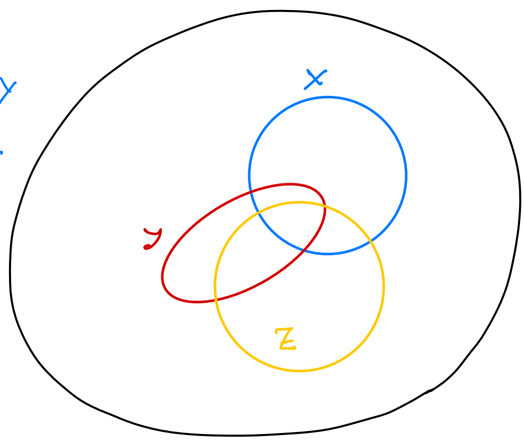
Experimental way to settle the debate between Einstein and quantum mechanics over the reality of pre-existing properties.

Nobel prize 2022

Bell inequality

$$P(x, y) \le P(y, z) + P(x, \neg z)$$

Exercise: Show the inequality with Van diagram.



* Can this logical inequality be violated?

Yes! in QM. How?

How should one interpret the violation of Bell's inequality in QM?

will be discussed in Chapter 6.

1.5. Projects for students

- your own choosing!
- Quantum Zeno-effect
- Delayed choice quantum eraser
- Decoherence
- Gravitation as the cause of the collapse of the wave-function, Penrose
- The Role of Consciousness in Collapse of the wave function, Wigner, von Neumann
- Time-Symmetric Interpretation (Aharonov, Vaidman) Weak measurement
- Super-determinism