The Pilot-Wave Theory

Rewritten from Travis Norsen

Ignorance Interpretation

Statistical quantum mechanics does not pretend to describe the individual system and its evolution in time completely… it's unavoidable to look elsewhere for a complete description of the individual system.

A. Einstein, Reply to criticisms, 1949

Wave Particle duality

While the founding fathers agonized over the question

'particle' *or* 'wave'

de Broglie in 1925 proposed the obvious answer

'particle' *and* 'wave'.

Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in [a] screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored. Of the founding fathers, only Einstein thought that de Broglie was on the right lines. Discouraged, de Broglie abandoned his picture for many years. He took it up again only when it was rediscovered, and more systematically presented, in 1952, by David Bohm. .... There is no need in this picture to divide the world into 'quantum' and 'classical' parts. For the necessary 'classical terms' are available already for individual particles (their actual positions) and so also for macroscopic assemblies of particles [4].

J.S. Bell, Six possible worlds of quantum mechanics, Speakable and Unspeakable in Quantum Mechanics

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi \qquad (1)$$

$$p = \frac{h}{\lambda} = \hbar k \qquad\qquad v = \frac{p}{m} = \frac{\hbar}{m}k$$

Write the wave function in the polar form:

$$\Psi(x,t) = R(x,t)e^{iS(x,t)}$$

$$v = \frac{\hbar}{m}\frac{\partial S}{\partial x}$$

For the plane-wave type solution: $S(x,t) = kx$

$$\frac{dX(t)}{dt} = \frac{\hbar}{m}\left.\frac{\partial S(x,t)}{\partial x}\right|_{x=X(t)} \qquad (2)$$

$X(t)$ Actual position of the particle

The so called hidden variable!

this can be equivalently re-written (in terms of the wave function itself) as follows:

$$\frac{dX(t)}{dt} = \frac{\hbar}{m}\,\mathrm{Im}\left[\frac{\left(\frac{\partial \Psi}{\partial x}\right)}{\Psi}\right]\Bigg|_{x=X(t)}$$

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V(x,t)\Psi \tag{3}$$

$$-i\hbar\frac{\partial \Psi^*}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi^*}{\partial x^2} + V(x,t)\Psi^* \tag{4}$$

Multiply eq. 3 with Ψ^* and eq. 4 with Ψ, then subtract:

$$\frac{\partial}{\partial t}|\Psi|^2 = -\frac{\partial}{\partial x}\left[\frac{i\hbar}{2m}\left(\Psi\frac{\partial}{\partial x}\Psi^* - \Psi^*\frac{\partial}{\partial x}\Psi\right)\right]$$

This has the form of the continuity equation:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}j$$

Probability density current: $j = \frac{i\hbar}{2m}\left(\Psi\frac{\partial}{\partial x}\Psi^* - \Psi^*\frac{\partial}{\partial x}\Psi\right)$

Analogous to Electrodynamics $\qquad \vec{v} = \dfrac{\vec{j}}{\rho}$

$$\vec{v} = \frac{\vec{j}}{\rho} = \frac{i\hbar}{2m}\,\frac{\Psi\vec{\nabla}\Psi^* - \Psi^*\vec{\nabla}\Psi}{\Psi^*\Psi} \tag{4}$$

Since
$$\frac{\left(\Psi\frac{\partial}{\partial x}\Psi^* - \Psi^*\frac{\partial}{\partial x}\Psi\right)}{-2i} = \mathrm{Im}\left(\Psi^*\frac{\partial}{\partial x}\Psi\right)$$

Eq. 4 becomes:

$$v = \frac{\hbar}{m}\,\frac{\mathrm{Im}\left(\Psi^*\frac{\partial}{\partial x}\Psi\right)}{\Psi^*\Psi} = \frac{\hbar}{m}\mathrm{Im}\left[\frac{\left(\frac{\partial\Psi}{\partial x}\right)}{\Psi}\right]$$

Which is the same equation postulated by Bohm for the velocity of particle!

It can be shown that:

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left[\underbrace{v(x,t)}_{\frac{j}{|\psi|^2}}\,\underbrace{P(x,t)}_{|\psi|^2}\right]$$

If $P(x,0) = |\Psi(x,0)|^2$ then it will stay $P(x,t) = |\Psi(x,t)|^2$

Particle in a Box (PIB) with Pilot-wave theory

Stationary states: $$\Psi(x,t) = \psi_1(x)e^{-iE_1t/\hbar}$$

where $\psi_1(x) = \sqrt{\frac{2}{L}}\sin(\pi x/L)$ is the first eigenstate with ground level energy E1

The complex phase of the wave function is: $S(x,t) = -iE_1t/\hbar$

This doesn't depend on x at all, so the particle velocity, according to Bohm's equation is zero. The particle, that is, just sits there at rest. This, as it turns out, is characteristic of so-called stationary states, which are indeed aptly named according to this theory.

To see some non-trivial dynamics in the particle-in-a-box system, we need only let the quantum state be a superposition of energy eigenstates. For example:

$$\begin{aligned}\Psi(x,t) &= \frac{1}{\sqrt{2}}\left[\psi_1(x)e^{-iE_1t/\hbar} + \psi_2(x)e^{-iE_2t/\hbar}\right] \\ &= \frac{1}{\sqrt{L}}\left[\sin(\pi x/L)e^{-i\omega_1 t} + \sin(2\pi x/L)e^{-i\omega_2 t}\right]\end{aligned}$$

Then particle velocity as a function of its position is:

$$\frac{dX(t)}{dt} = \frac{\hbar}{m}\mathrm{Im}\left[\frac{\frac{\pi}{L}\cos\left(\frac{\pi x}{L}\right)e^{-i\omega_1 t} + \frac{2\pi}{L}\cos\left(\frac{2\pi x}{L}\right)e^{-i\omega_2 t}}{\sin\left(\frac{\pi x}{L}\right)e^{-i\omega_1 t} + \sin\left(\frac{2\pi x}{L}\right)e^{-i\omega_2 t}}\right]_{x=X(t)}$$

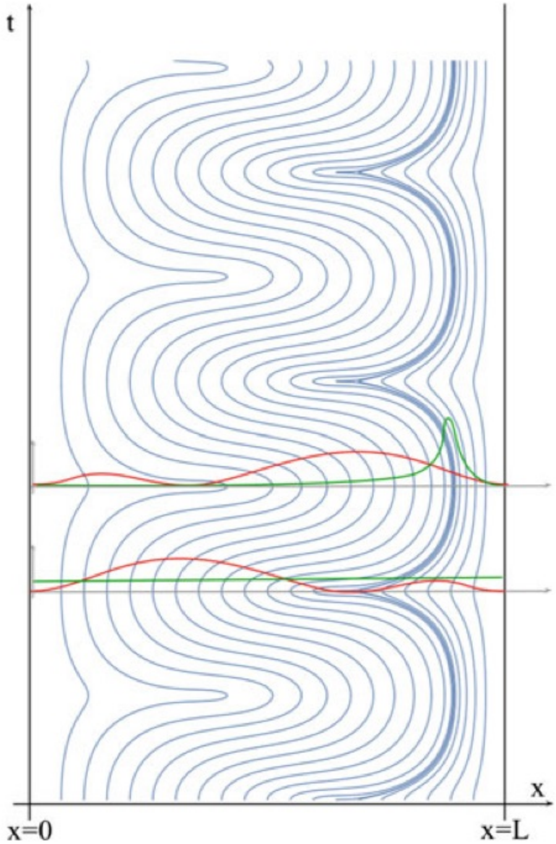


Fig. 7.1 The *blue curves* are a set of possible worldlines for a particle-in-a-box with wave function an equally-weighted superposition of the $n = 1$ and $n = 2$ energy eigenstates. (This is a space-time diagram, with the horizontal axis being the position x within the box, and the vertical axis representing the time t.) Note that at $t = 0$ the example trajectories are equally-spaced across the box. If we think of this as an ensemble of trajectories, we would say that the initial distribution $P(x)$ is constant. But then the distribution at later times is not constant, as illustrated by the rather extreme "clumping up" of the world lines. The distribution P is graphed, as a function of x, at two different times: see the *green curves* that live on the grey axes, whose vertical location is meant to indicate the time. The associated *red curves* show what $|\Psi|^2$ looks like at these same times

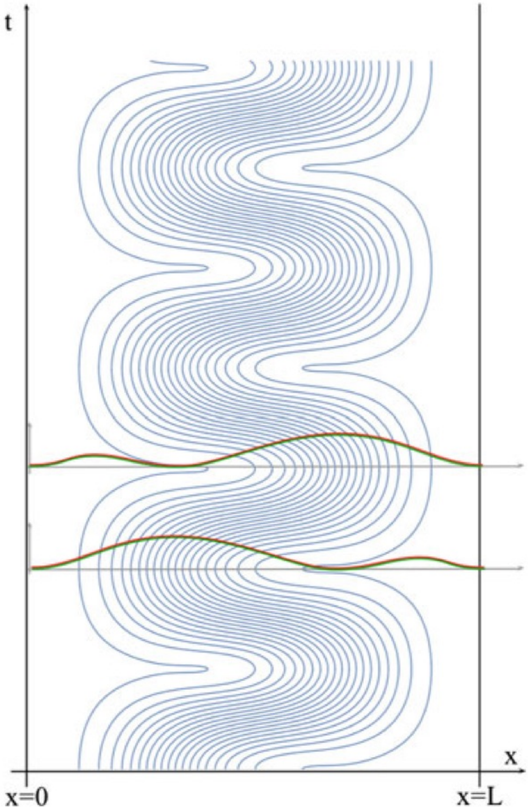


Fig. 7.2 Same as Fig. 7.1 but for an ensemble of initial positions $X(0)$ that are distributed with $P(x,0) = |\Psi(x,0)|^2$. This illustrates the "equivariance" property discussed in the previous section: if the positions of particles in the ensemble are $|\Psi|^2$-distributed at $t = 0$, then they will remain $|\Psi|^2$-distributed for all time. So the *green curves*(indicating P) and the *red curves*(indicating $|\Psi|^2$) coincide at all times here, unlike the situation depicted in the previous figure

Other single particle examples

Gaussian wave packet:

Initial state $$\Psi(x,0) = Ne^{-x^2/4\sigma^2}$$

Evolution in time $$\Psi(x,t) = N(t)\, e^{-x^2/4(\sigma^2+i\hbar t/2m)}$$

Written in polar form:

$$\Psi(x,t) = N(t)\,\exp\left[\frac{-x^2\sigma^2}{4(\sigma^4+\hbar^2t^2/4m^2)}\right]\exp\left[\frac{ix^2\hbar t}{8m(\sigma^4+\hbar^2t^2/4m^2)}\right].$$

we can identify the complex phase S(x, t) of the wave function as this, ignoring the contribution of N(t) to the complex phase:

$$S(x,t) = \frac{x^2\hbar t}{8m(\sigma^4+\hbar^2t^2/4m^2)}$$

then we have $$\frac{dX(t)}{dt} = X(t)\,\frac{t}{t^2+4m^2\sigma^4/\hbar^2}$$

It is not hard to show that this differential equation is solved by:

$$X(t) = X_0\left(1+\frac{t^2}{4m^2\sigma^4/\hbar^2}\right)^{1/2} \quad \text{or} \quad \left(\frac{X(t)}{X_0}\right)^2 - \left(\frac{t}{2m\sigma^2/\hbar}\right)^2 = 1$$

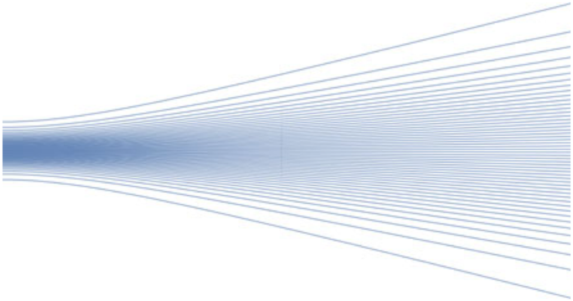


Fig. 7.3 Representative sample of particle trajectories for a spreading Gaussian wave packet. Here time runs to the *right* and x runs vertically (so it is a space-time diagram turned sideways). Or one can, in the spirit of Fig. 2.6, consider replacing t with a second spatial coordinate, and hence think of the lines as showing the trajectories through space that particles would follow downstream of a single Gaussian slit. That is, the figure can be understood as showing the trajectories that particles would follow when being guided by a diffracting wave function. Note that the distribution of initial particle positions $X(0)$ here is given by $|\Psi(x, 0)^2|$, so (by "equivariance") the trajectories spread out from one another so as to keep $P = |\Psi|^2$ for subsequent times

Superposition of two Gaussian wave packets at double slit

$$\Psi(x, t) = N(t)\left[e^{-\frac{(x-a)^2}{4(\sigma^2+i\hbar t/2m)}} + e^{-\frac{(x+a)^2}{4(\sigma^2+i\hbar t/2m)}}\right]$$

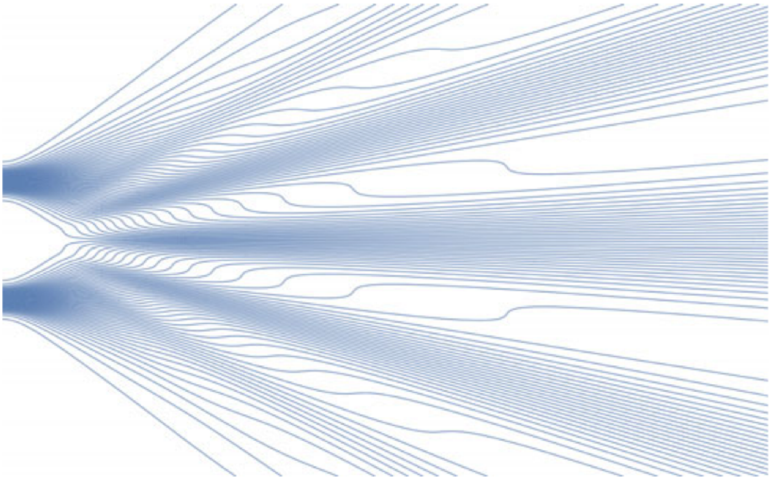


Fig. 7.4 Representative sample of particle trajectories for the case of two initially-separated Gaussian wave-packets. As in the previous figure, this is technically a space-time diagram turned sideways – but one may also legitimately think of it as showing the trajectories, through space, of particles which have just emerged, moving to the *right*, through a double- (Gaussian) slit screen. This type of image, of the particle trajectories for the double-slit experiment according to the pilot-wave theory, was first presented in Ref. [5] and has become iconic for the pilot-wave theory because it captures so clearly, in a picture, how the theory explains the (otherwise) puzzling wave-particle-duality. The discrete flashes on the detection screen correspond to places where (literal, pointlike) particles collide with the screen; but the highly non-classical motion of the particles is influenced by the accompanying pilot-wave such that the particle trajectories bunch up around points of constructive interference. An ensemble of such trajectories (with suitably random initial conditions) will therefore perfectly account (in, to use Bell's phrase, "a clear and ordinary way" [4]) for the type of statistical interference pattern we saw in Fig. 2.8

Measurement in Pilot-wave theory

Review of Measurement Problem in PIB example:

Interaction energy $\hat{H}_{int} = \lambda \hat{H}_x \hat{p}_y$

If the particle is in an eigenstate: $\Psi(x, y, 0) = \psi_n(x)\phi(y)$

After the measurement time t, the state of the PIB-Pointer is:

$$\Psi(x, y, t) = \psi_n(x)\phi(y - \lambda E_n t)$$

No problem there.

But if the particle was in a superposition state: $\Psi(x, y, 0) = \left[\sum_i c_i \psi_i(x)\right]\phi(y)$

then it infects the measuring apparatus with its superposition!
Schrödinger's cat

$$\Psi(x, y, t) = \sum_i c_i \psi_i(x)\phi(y - \lambda E_i t)$$

Also according to the pilot wave theory the PIB-Pointer wave function ends up in an entangled superposition state as stated above.

But there is no measurement problem in pilot wave theory! [Why?]

According to the pilot-wave theory the wave function alone doesn't provide a complete description of the physical situation. There is, in addition, the actual position X(t) of the PIB and -crucially here- the actual position Y(t) of the pointer.

$$\frac{dX(t)}{dt} = \frac{j_x}{|\Psi|^2} \qquad \frac{dY(t)}{dt} = \frac{j_y}{|\Psi|^2}$$

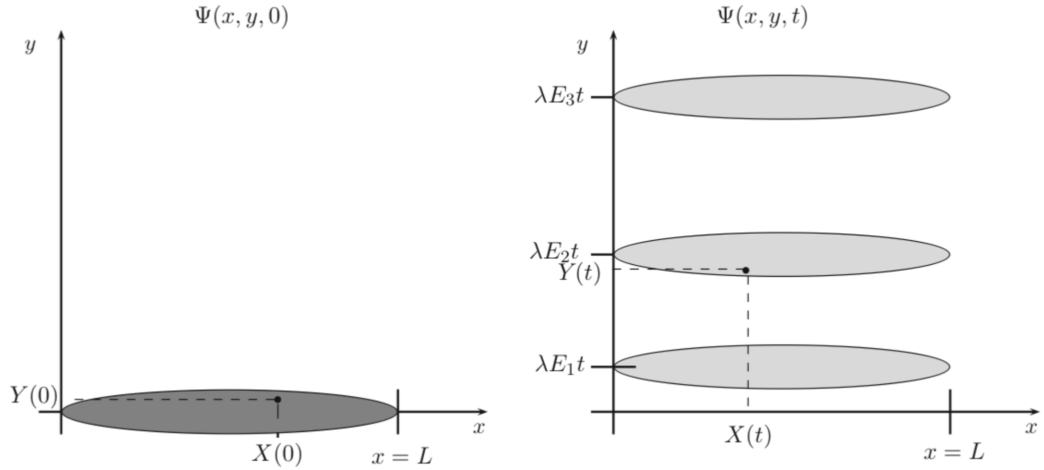


Fig. 7.5 The graph on the *left highlights* (in *dark gray*) the region of the two-dimensional configuration space where $\Psi(x, y, 0)$ has support. Later, at time t, the wave function has split apart into several non-overlapping "islands". This is depicted in the graph on the *right*. The simultaneous presence of all these islands constitutes, for orthodox quantum mechanics, the measurement problem. But for the pilot-wave theory, the actually-realized outcome of the measurement is not to be found in the wave function, but rather in the final position of the pointer. And this, in the pilot-wave theory, will be some one (random but perfectly definite) value, indicated here by the vertical position $Y(t)$ of the *dot* which represents the actual configuration point (X, Y). The indicated $Y(t)$ is in the support of the $n = 2$ branch of the wave function – i.e., $Y(t)$ is approximately $\lambda E_2 t$ – so we would say in this case that the energy measurement had the outcome $E = E_2$. Note that the outcome might have been different had the (random) initial positions $X(0)$ and $Y(0)$ been different

Measurement in Bohmian mechanics

PIB-Pointer wave function

$$\Psi(x, y, t) = \sum_i c_i \psi_i(x)\phi(y - \lambda E_i t)$$

PIB wave function

$$\chi(x, t) \sim \sum_i c_i \psi_i(x)\phi(Y(t) - \lambda E_i t)$$

Initial PIB wave function

$$\chi(x, 0) \sim \sum_i c_i \psi_i(x)\,\phi\,(Y(0)) \sim \sum_i c_i \psi_i(x)$$

In long t limit where the measurement is done

$$\chi(x, t) \sim \sum_i c_i \psi_i(x)\phi(Y(t) - \lambda E_i t) \approx c_n \psi_n(x)\phi(Y(t) - \lambda E_n t)$$

$$\chi(x, t) = \psi_n(x)$$

Contexuality

Do pre-measurement values exist according to the pilot wave theory?

Yes and no and it depends!

Location of particles in space is real and they exist independent of our measurements. Therefore their detection reveals the pre-measurement values.

Momentum of particles also have real pre-measurement values. But its measurement doesn't necessarily reveal the pre-measurement value but changes it during measurement interaction.

Other physical properties like Energy, Spin, … don't have any pre-measurement values!

In pilot-wave theory the measurement outcomes depend on initial conditions (actual location of particles and pointers, initial wave function) and the measurement method of the quantity as well.

In pilot-wave theory it's not weird that a quantity such as spin didn't exist prior to the measurement.

The "no hidden variables" theorems, invariably apply only to "non-contextual" hidden variables theories. That is why those theorems do not in an any sense rule out the pilot-wave theory.

The Many-Particle Theory and Nonlocality

The pilot-wave theory is non-local in the following sense: the velocity of each particle, at a given instant, depends on the instantaneous positions of all other particles (at least when there is entanglement).

Example: two-particle system with wave function $\Psi(x_1, x_2, t)$

The velocity of particle 1:

$$v_1(t) = \frac{dX_1(t)}{dt} = \frac{\hbar}{m_1} \operatorname{Im}\left[\frac{\left(\frac{\partial\Psi(x_1, X_2(t), t)}{\partial x_1}\right)}{\Psi(x_1, X_2(t), t)}\right]\Bigg|_{x_1 = X_1(t)}$$

Example: two "particle-in-a-box" sub-systems that are well-separated in space (so the origins of the x1 and x2 coordinate systems are far apart from each other) in the entangled state:

$$\Psi(x_1, x_2, t) = \frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2) + i\,\psi_2(x_1)\psi_1(x_2)\right] e^{-i(E_1+E_2)t/\hbar}$$

where, as usual, $\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$

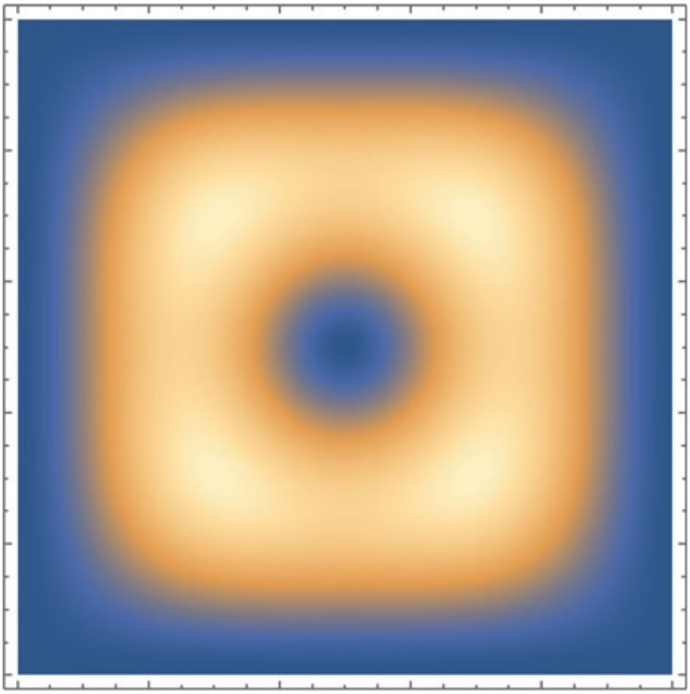


Fig. 7.6 Density plot of $|\Psi|^2$ in configuration space, with Ψ given by Eq. (7.55). The horizontal axis is x_1 and the vertical axis is x_2; there is a node in the *center* (where $\Psi = 0$) and then a "ring" where $|\Psi|^2$ is *large*. Note, though, that even though $|\Psi|^2$ is independent of time, the probability $|\Psi|^2$ is not stationary, but is instead flowing, clockwise, around the ring, like in a whirlpool

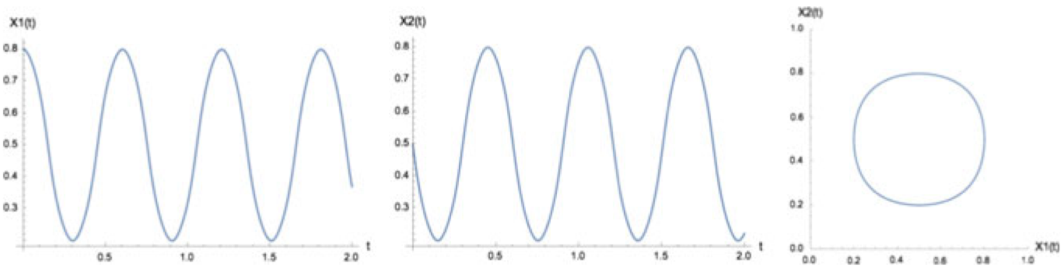


Fig. 7.7 The *left* and *center panels* show how the positions of the two particles ($X_1(t)$ and $X_2(t)$) vary with time: each particle essentially oscillates back and forth inside its box. The *right panel* shows the trajectory of the configuration point $\{X_1(t), X_2(t)\}$ through configuration space. (The trajectory is a closed clockwise loop.)

> *proof* that *any* hidden variable account of quantum mechanics *must* have this extraordinary character.[*] It would therefore be interesting, perhaps, to pursue some further 'impossibility proofs', replacing the arbitrary axioms objected to above [namely, "non-contextuality"] by some condition of locality, or of separability of distant systems [13].

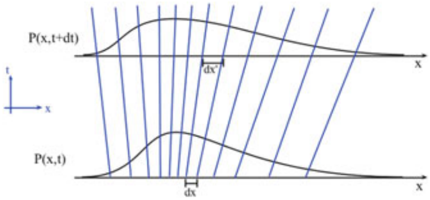
The "[$*$]" points to a footnote which was added before the delayed publication of the paper: "Since the completion of this paper such a proof has been found: J.S. Bell, Physics 1, 195, [1964]". That is, between the completion of this first paper in 1964, and its publication in 1966, Bell had already discovered and published the answer to his own question: would it be possible to construct a hidden variable completion of QM, with all of the virtues of the pilot-wave theory, but without the troubling non-local character?

His answer is the subject of Chap. 8.

Projects:

7.1 Show that Eq. (7.6) really is equivalent to Eq. (7.5).

7.2 Show that the probability distribution $P(x, t)$ for an ensemble of particles moving with velocities $v(x, t)$ should satisfy Eq. (7.20). Hint: argue, based on this picture



that all the trajectories (shown in the figure as blue lines on a space-time diagram) in dx at time t will be in dx' at time $t + dt$, i.e., $P(x + v(x, t)dt, t + dt)dx' = P(x, t)dx$. This can (with some additional work) then be shown to be equivalent to

$$\frac{\partial P(x, t)}{\partial t} = -\frac{\partial}{\partial x}\left[P(x, t)v(x, t)\right]. \tag{7.60}$$

7.3 Work through the derivation of Eq. (7.12) – the quantum continuity equation for a particle in three dimensions – from the time-dependent Schrödinger equation, and thereby confirm the expression in Eq. (7.14) for the quantum probability current.

7.4 Show that, indeed, Eq. (7.17) is equivalent to the earlier expressions for the particle velocity in the pilot-wave theory.

7.5 Confirm that Eq. (7.30) really solves Eq. (7.29).

7.6 Massage Eq. (7.32) into polar form. Let Mathematica numerically solve the differential equation $\frac{dX}{dt} = \frac{\hbar}{m}\frac{\partial S}{\partial x}$ to recreate trajectories like the ones shown in Fig. 7.4.

7.7 For the toy model of a measurement discussed in Sect. 7.4, Schrödinger's equation reads

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi \tag{7.61}$$

with $\hat{H} = \lambda\hat{H}_x\hat{p}_y$, where, in turn, $\hat{H}_x = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$ and $\hat{p}_y = -i\hbar\frac{\partial}{\partial y}$. Show that the x- and y-components of the quantum probability current can be written

$$j_x = -\frac{\lambda\hbar^2}{m}\text{Re}\left[\Psi^*\frac{\partial}{\partial x}\frac{\partial}{\partial y}\Psi\right] \tag{7.62}$$

and

$$j_y = \frac{\lambda\hbar^2}{m}\frac{\partial\Psi^*}{\partial x}\frac{\partial\Psi}{\partial x} \tag{7.63}$$

respectively. That is, show that the Schrödinger equation implies the continuity equation

$$\frac{\partial|\Psi|^2}{\partial t} = -\frac{\partial j_x}{\partial x} - \frac{\partial j_y}{\partial y} \tag{7.64}$$

with j_x and j_y as given above.

7.8 Use the results of the previous Project to argue that, in the pilot-wave theory, the velocities of the two particles involved in the toy model of measurement are given by

$$\frac{dX}{dt} = -\frac{\lambda\hbar^2}{m}\text{Re}\left[\frac{\frac{\partial}{\partial x}\frac{\partial}{\partial y}\Psi}{\Psi}\right] \tag{7.65}$$

and

$$\frac{dY}{dt} = \frac{\lambda\hbar^2}{m}\left|\frac{\partial\Psi/\partial x}{\Psi}\right|^2. \tag{7.66}$$

Let Mathematica numerically solve these differential equations to find some example trajectories $X(t)$, $Y(t)$. Use the known solution of Schrödinger's equation for this problem:

$$\Psi(x, y, t) = \sum_i c_i\psi_i(x)\phi(y - \lambda E_i t). \tag{7.67}$$

7.9 Use your Mathematica program from the previous Project to demonstrate the "contextuality" of energy measurements in the pilot-wave theory. In particular, find specific initial conditions that lead to *different* outcomes for the energy measurement for different values of λ. (You can do this by trial and error: just pick some random values for $X(0)$ and $Y(0)$ and then fiddle with the value of λ. You probably won't have to try too many different values of λ before you find a couple of values that produce distinct values of $E_n \approx \frac{Y(t)}{\lambda t}$.)

7.10 Show that Eq. (7.48) really solves the (time-dependent) Schrödinger equation, with $V(x)$ given by Eq. (7.45), as long as the packet width σ and the frequency ω are related as in Eq. (7.47).

7.11 Calculate j_{x_1} and j_{x_2} for the wave function in Eq. (7.55).

7.12 In the passage quoted in Sect. 7.7, Heisenberg refers to "some strange quantum potentials introduced ad hoc by Bohm". This is a reference to a slightly different formulation of the pilot-wave theory, in terms of which Bohm presents the theory in his 1952 papers. To see how this works, take a time derivative of Eq. (7.6) to derive an expression for the *acceleration* of the particle. It is important here that the right hand side of Eq. (7.6) depends on time in two different ways, so one must use the "convective derivative" $\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{dx}{dt}\frac{\partial}{\partial x}$. If all goes well you should be able to write the equation describing the motion of the particle in the somewhat more Newtonian-mechanical-like form,

$$ma = -\frac{\partial}{\partial x}(V + Q) \tag{7.68}$$

where V is the regular ("classical") potential energy function (which appears in Schrödinger's equation) and then Q is a new, so-called "quantum potential" which depends on the structure of the wave function ψ. Find the expression for Q. (It can be expressed most simply in terms of the R in $\psi = Re^{iS}$.) Think about how to understand, from this more classical perspective on the motion of the particle, how (for example) the electron particle in a ground-state Hydrogen atom remains at rest.

7.13 The previous Project might suggest that, in addition to the kinetic energy $K = \frac{1}{2}mv^2$ and classical potential energy $V(x)$, a particle in the pilot-wave theory also possesses some "quantum potential energy", Q. Would the inclusion of this "quantum potential energy" make it possible to regard the measurement of the energy of a particle as revealing a pre-existing energy value? In other words, does the possibility of re-formulating the theory in this more Newtonian-mechanical-like way undermine our conclusion that energy is, in the pilot-wave theory, contextual?

7.14 Suppose we define the total energy of a particle in the pilot-wave theory as $E = K + V + Q$ as suggested in the previous Project. Is the total energy E of a particle conserved according to the theory?

7.15 One of Heisenberg's criticisms of Bohm's theory is that "[i]n measurements of the position of the particle, Bohm takes the ordinary interpretation of the experiments as correct; in measurements of the velocity he rejects it." Heisenberg here means that, in the pilot-wave picture, position is non-contextual whereas velocity is contextual. Heisenberg seems to think that this is the result of a choice and is therefore arbitrary and unbelievable. But we have shown in the Chapter that the contextuality of (for example) momentum and energy is not a choice at all, but simply a consequence of the basic dynamical postulates of the theory. Complete the rebuttal of Heisenberg's criticism by showing that position measurements just do, according again to the dynamical postulates, reveal pre-existing position values. (Hint: consider a particle with wave function $\psi_0(x)$ whose position is to be measured using an apparatus whose pointer has an initial wave function $\phi(y)$ and interacts with the particle according to $\hat{H}_{int} = \lambda \hat{x} \hat{p}_y$. Show that the x- and y-components of the quantum probability current can be taken to be $j_x = 0$ and $j_y = \lambda x \Psi^* \Psi$ so that $dX/dt = 0$ and $dY/dt = \lambda X$. This implies that the final displacement of the pointer is proportional to X, the actual position of the particle whose position is being measured.)

7.16 Read through Bohm's 1952 papers [3] and report on anything you find interesting or surprising.

7.17 Read through Ref. [14], "The pilot-wave perspective on quantum scattering and tunneling," and summarize its main points.

7.18 Read through Ref. [15], "The pilot-wave perspective on spin," and summarize its main points. In particular, explain in detail how the pilot-wave theory accounts for the EPR-Bohm correlations. Bell says that the theory resolves the EPR paradox "in the way which Einstein would have liked least". What exactly does he mean?

7.19 Can the pilot-wave theory be diagnosed as "nonlocal" using Bell's formulation of locality (or the slightly modified formulation) from Chap. 1? How about using the related necessary condition for locality that we developed in Chap. 5?

7.20 In the text, the non-locality of the pilot-wave theory is explained in terms of an entangled two-particle state (with a measurement of one of the particles non-locally affecting the motion of the other, distant particle). But we saw in Chap. 4 that textbook quantum theory is already apparently non-local in the simpler, single-particle "Einstein's boxes" scenario. Is any non-locality involved in the pilot-wave theory's account of "Einstein's boxes"? Explain.

7.21 Recall the passage quoted in Sect. 6.6, in which (the textbook author) David Griffiths explains three frequently-encountered attitudes toward quantum mechanics: the "realist" position, the "orthodox" position, and the "agnostic" position. Would Griffiths classify the pilot-wave theory as "realist"? Note that your answer will depend on whether or not you think he intends what he says about a position measurement, by way of defining what he means by "realist", to apply *just* to position measurements, or instead to apply more generally to measurements of any property. Why do you think Griffiths isn't more explicit about this issue, namely, whether, to count as "realist", a theory should merely say that position measurements reveal pre-existing position values, or instead must say that a measurement of *any* quantity must reveal its pre-existing value?

7.22 In the passage quoted in Sect. 6.6, Griffiths seems to define "realism" as meaning that a theory posits specifically non-contextual hidden variables (at least for position). The pilot-wave theory would count as "realist" in this sense (if we interpret this notion of "realism" as requiring non-contextual hidden variables *only* for positions... obviously the pilot-wave theory would *not* be "realist" if that is taken to require non-contextual hidden variables for not only position, but also momentum, energy, etc.). But there would seem to be a more basic